bered curves. This explains why only the lower ordered leaky poles are important for a thin slab when the steepest descent path is adopted. A comparison of Fig. 2, which shows the steepest descent paths for various observation angles, with Figs. 4 to 7 demonstrates that many leaky wave poles can be captured by the deformation, depending on the observation angle and physical parameters of the slab. The residues of the leaky wave poles are highly attenuated in the far field, as claimed by Fang and Chow [4]. In fact, they completely ignored the residues of the leaky poles for separation distance $k_0 r > 2\pi$ and observation angle $\theta = \pi/2$, i.e., on the interface. However, it is clear from [15] that for some values of the dielectric constant and height, the closeness of the leaky wave poles to the steepest descent path must still be taken into account, even for $k_0 r = 100$ and $0 < \theta < \pi/2$. Although the physical parameters employed in [4] and [15] are different, our purpose is to emphasize that care must be taken when ignoring the residues of the leaky wave poles. Moreover, without knowing the locations or the loci of the leaky wave poles, this cannot be done satisfactorily.

III. Concluding Remarks

A simple numerical procedure for finding the loci of TE and TM leaky wave poles as the frequency or the thickness of the slab varies is presented. These loci provide important information when the integration path of the Sommerfeld integral for the grounded dielectric slab problem is deformed into the "improper" sheet of the Riemann surface. The accuracy of the loci has been checked extensively against contour plots of expressions (1) and (2) with the B's as parameters.

Acknowledgment

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References

[1] L. B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves.* Englewood Cliffs, NJ: Prentice Hall, 1973.
[2] E. H. Newman and D. Forrai, "Scattering from a microstrip patch," *IEEE Trans. Antennas Propagat.*, vol. AP-35, pp. 245–251, Mar. 1987.
[3] K. Michalski and D. Zheng, "Modeling antennas and scatterers of arbitrary shape embedded in layered dielectric media," Tech. Rep., Dept. Elec. Eng., Texas A&M University, Nov. 1989.
[4] D. G. Fang and Y. L. Chow, "The Green's function along the microstrip substrate from a horizontal magnetic dipole," *J. Appl. Phys.*, vol. 54, pp. 33–38, Jan. 1983.
[5] T. K. Sarkar, "Analysis of arbitrarily oriented thin wire antennas over a plane imperfect ground," *Arc. Elek. Übertragung.*, vol. 31, pp. 449–457, 1977.
[6] A. Banõs, *Dipole Radiation in the Presence of a Conducting Half-Space.* New York: Pergamon Press, 1966.
[7] S. Barkeshli, "Efficient approaches for evaluating the planar microstrip Green's function and its applications to the analysis of microstrip antennas," Ph.D. dissertation, Dept. Elec. Eng., Ohio State Univ., 1988.
[8] M. Marin, S. Barkeshli, and P. H. Pathak, "On the location of proper and improper surface wave poles for the grounded dielectric slab," *IEEE Trans. Antennas Propagat.*, vol. 38, pp. 570–573, Apr. 1990.
[9] R. E. Collin, *Field Theory of Guided Waves.* New York: McGraw-Hill, 1960.
[10] J. R. Mosig and F. E. Gardiol, "A dynamical radiation model for microstrip structures," in *Advances in Electronics and Electron Physics*, vol. 59. New York: Academic Press, 1982, pp. 139–237.
[11] G. N. Tsandoulas, "Excitation of a grounded dielectric slab by a horizontal dipole," *IEEE Trans. Antennas Propagat.*, vol. AP-17, pp. 156–161, Mar. 1969.
[12] R. F. Harrington, *Time-Harmonic Electromagnetic Fields.* New York: McGraw-Hill, 1961.
[13] T. Tamir and A. A. Oliner, "Guided complex waves: Part 1," *Proc. Inst. Elec. Eng.*, vol. 110, pp. 310–324, Feb. 1963.
[14] K. G. Budden, *The Wave-Guide Mode Theory of Wave Propagation.* Englewood Cliffs, NJ: Prentice-Hall, 1961.
[15] G. D. Bernard and A. Ishimaru, "On complex waves," *Proc. Inst. Elec. Eng.*, vol. 114, pp. 43–49, Jan. 1967.

Dispersion Characteristics of Strip Dielectric Waveguides

Kin S. Chiang

Abstract —**A simple and accurate dispersion relation is derived for the guided modes of a strip dielectric waveguide. This relation shows explicitly the effect of the width of the waveguide and involves only the solution for a single three-layer slab waveguide. It is discovered that there always exists a strip waveguide with a specific aspect ratio in which the E^x_{mn} and E^y_{mn} modes propagate at the same phase velocity.**

I. Introduction

A strip dielectric waveguide of the type shown in Fig. 1(a) is a basic and important wave-guiding structure and serves as a building block in many transmission devices at millimeter-wave and optical frequencies [1]–[4]. While exact analytical solutions are not available, this waveguide has been analyzed by various semianalytical and numerical methods, which include the effective-index method [1], [2], [5], [6], Marcatili's method [7], the mode-matching method [8], the finite-element method [9], [10], the finite-difference method [11], [12], and the weighted-index method [13]. However, many of these methods [8]–[13] require massive computation and the physical properties of the waveguide are not apparent in such analyses.

In this paper a simple approximate expression is derived to describe explicitly the dispersion characteristics of the guided modes of a strip waveguide. The accuracy of this expression is confirmed by comparison with results from other methods. The use of this expression should significantly simplify the study and design of strip waveguides.

II. Analysis and Results

We consider the embossed wave-guiding structure as shown in Fig. 1(a), which is commonly referred to as a strip waveguide, an insulated image guide, or a special type of rib waveguide. We adopt here the optics terminology by denoting n_1, n_2, and n_3 ($n_1 > n_2 > n_3$) as the refractive indexes of the strip, the substrate, and the superstrate (usually air), respectively. The strip has a height $2b$ and a width $2a$ and the substrate and the superstrate are assumed unbounded. The guided mode of such a waveguide can be designated as the E^i_{mn} mode, which has a predominant electric field component in the i ($i = x$ or y) direction with $m-1$ and $n-1$ ($m,n \geqslant 1$) field zeros in the x and y directions, respectively. The refractive-index profile of the waveguide is characterized by two relative refractive-index steps, Δ_1 and Δ_2, defined by

$$\Delta_1 = \frac{n_1^2 - n_2^2}{2n_1^2} \tag{1}$$

and

$$\Delta_2 = \frac{n_1^2 - n_3^2}{2n_1^2}. \tag{2}$$

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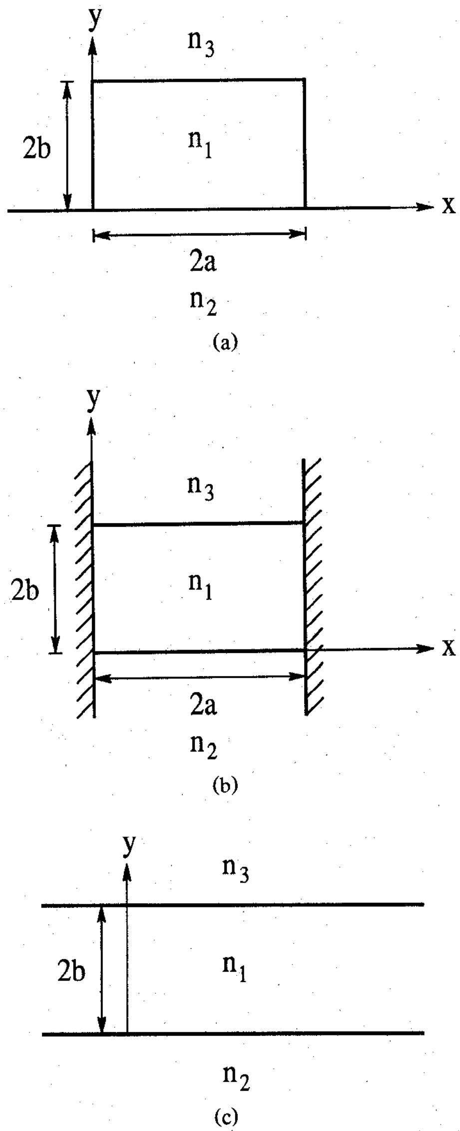

Fig. 1. Cross sections of (a) a strip waveguide, (b) a metal-clad waveguide, and (c) a three-layer slab waveguide.

In practice, the refractive-index difference between the strip and the substrate is much smaller than that between the strip and the superstrate, i.e., $\Delta_2 \gg \Delta_1$. The cutoff mode index (the mode index being the propagation constant of the mode divided by the free-space wavenumber) is set by the refractive index of the substrate. The mode field is well confined in the x direction even near cutoff, where the field penetrates significantly only into the substrate. The predominant transverse electric field, ψ_i, of the E^i_{mn} mode of the strip waveguide can thus be approximated by $\tilde{\psi}_i$, the field of the $\tilde{E}^i_{mn}$ mode of a metal-clad waveguide, shown in Fig. 1(b), which is obtained by replacing the areas at both sides of the strip with metal in which the field vanishes:

$$\psi_i(x,y) \simeq \tilde{\psi}_i(x,y) = \sin\frac{m\pi x}{2a}\hat{\psi}_i(y), \qquad 0 \leqslant x \leqslant 2a, \quad -\infty < y < +\infty. \quad (3)$$

Here $\hat{\psi}_i(y)$ is the transverse electric field of the TE_{n-1} $(i=x)$ or TM_{n-1} $(i=y)$ mode of a three-layer slab waveguide with a thickness $2b$, as shown in Fig. 1(c).

The strip waveguide is now treated as a perturbation of the metal-clad waveguide. A perturbation formula relating the propagation constants of two waveguides has been derived [14]. In our context, it is given by

$$\beta_i^2 = \tilde{\beta}_i^2 + \frac{\displaystyle\int_{-\infty}^{+\infty}\left[\left(\psi_i\frac{\partial\tilde{\psi}_i}{\partial x}\right)_{x=0} - \left(\psi_i\frac{\partial\tilde{\psi}_i}{\partial x}\right)_{x=2a}\right]dy}{\displaystyle\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\psi_i\tilde{\psi}_i\,dx\,dy} \quad (4)$$

where β_i and $\tilde{\beta}_i$ are the propagation constants of the E^i_{mn} modes of the strip waveguide and the metal-clad waveguide, respectively. In deriving (4), the boundary condition $\tilde{\psi}_i(0,y) = \tilde{\psi}_i(2a,y) = 0$ is used and the orthogonal transverse field components are neglected.

Although (3) is accurate for $0 < x < 2a$, it gives a zero value of ψ_i at $x=0$ and $x=2a$, which is not accurate enough for the evaluation of (4). Since $\Delta_2 \gg \Delta_1$, the field in the superstrate decays exponentially from the core–superstrate boundary according to $\exp[-(V/b)(\Delta_2/\Delta_1)^{1/2}r]$, where V is defined by (6) (see below) and r is the distance from the boundary. When the field is parallel to the boundary, the normal field gradient is continuous across the boundary. When the field is perpendicular to the boundary, however, it is the normal field gradient divided by the square of the refractive index that is continuous. For both cases, we can find the field value at the boundary from the normal field gradient evaluated in the core near the boundary:

$$\begin{aligned}\psi_i(0,y) &= (-1)^{m+1}\psi_i(2a,y)\\ &\simeq \frac{b}{V}\left(\frac{\Delta_1}{\Delta_2}\right)^{1/2}\left(\frac{\partial\tilde{\psi}_i}{\partial x}\right)_{x=0}(1-2\Delta_2 S_i)\\ &= \frac{m\pi b}{2aV}\left(\frac{\Delta_1}{\Delta_2}\right)^{1/2}(1-2\Delta_2 S_i)\hat{\psi}_i(y) \quad (5)\end{aligned}$$

with

$$V = bk\left(n_1^2 - n_2^2\right)^{1/2}. \quad (6)$$

V is the normalized frequency, k the free-space wavenumber, and the factor $1-2\Delta_2 S_i$ discriminates between the continuity conditions across the boundary for the two polarizations for which $S_x = 1$ and $S_y = 0$. Strictly speaking, (5) is accurate only when $0 < y < +\infty$. For the sake of simplicity, we assume that (5) is also valid when $-\infty < y \leqslant 0$. Putting (3) and (5) into (4), we find

$$\beta_i^2 = \tilde{\beta}_i^2 + \frac{m^2\pi^2}{2a^3V}\left(\frac{\Delta_1}{\Delta_2}\right)^{1/2}(1-2\Delta_2 S_i). \quad (7)$$

The propagation constant of the $\tilde{E}^i_{mn}$ mode of the metal-clad waveguide can be solved exactly by separation of variables:

$$\tilde{\beta}_i^2 = \hat{\beta}_i^2 - \frac{m^2\pi^2}{4a^2} \quad (8)$$

where $\hat{\beta}_i$ is the propagation constant of the TE_{n-1} $(i=x)$ or TM_{n-1} $(i=y)$ mode of the three-layer slab waveguide in Fig. 1(c). Substituting (8) into (7), we obtain

$$\beta_i^2 = \hat{\beta}_i^2 - \frac{m^2\pi^2}{4a^2}\left[1 - \frac{2b}{aV}\left(\frac{\Delta_1}{\Delta_2}\right)^{1/2}(1-2\Delta_2 S_i)\right]. \quad (9)$$

This is the desired expression for the dispersion characteristics of the strip waveguide. The polarization effects associated with the modes are fully taken into account in (9), which can be regarded as an approximate solution for the vector wave equation. When $\hat{\beta}_i = \hat{\beta}_x$ and $S_i = 0$ are used, (9) represents the dispersion relation for the scalar modes, which satisfy the scalar wave equation.

According to (9), we need to use only the well-known dispersion characteristics of a three-layer slab waveguide to determine the dispersion characteristics of the strip waveguide. Of course, the significance of (9) is not just the provision of an efficient way of calculating the propagation constants for the strip waveguide. The important property of (9) is that the effects of the first mode order, m, and the strip width, $2a$, are explicitly given. This property makes (9) particularly useful in waveguide optimization, since the width of the waveguide can be easily tailored to suit a particular application.

In general the E^x_{mn} and E^y_{mn} modes of a strip waveguide have different propagation constants; their difference is usually termed the modal birefringence. The condition for the model birefringence to vanish can be immediately obtained from (9):

$$\hat{\beta}_x^2 - \hat{\beta}_y^2 = \frac{m^2\pi^2 b(\Delta_1\Delta_2)^{1/2}}{a^3 V}. \tag{10}$$

Since the propagation constant of the TE_{n-1} mode of a three-layer slab waveguide is always larger than that of the TM_{n-1} mode, i.e., $\hat{\beta}_x > \hat{\beta}_y$, there always exists an aspect ratio a/b for the strip such that (10) is satisfied. To find this particular aspect ratio, (10) can be rearranged as

$$\frac{a}{b} = \left[\frac{m^2\pi^2(\Delta_1\Delta_2)^{1/2}}{b^2 V(\hat{\beta}_x^2 - \hat{\beta}_y^2)}\right]^{1/3}. \tag{11}$$

The required aspect ratio can thus be calculated from (11) for a given value of V, provided that V is larger than the cutoff values for both the E^x_{mn} and E^y_{mn} modes of the strip waveguide. To estimate the magnitude of the required aspect ratio, we derive an asymptotic expression for $\hat{\beta}_x^2 - \hat{\beta}_y^2$ by treating the three-layer slab waveguide in Fig. 1(c) as a perturbation of a metal-clad slab waveguide by replacing the superstrate with metal. Following steps similar to those that led to (7), we obtain

$$\hat{\beta}_x^2 - \hat{\beta}_y^2 = \frac{n^2\pi^2(\Delta_1\Delta_2)^{1/2}}{2b^2 V} \tag{12}$$

It should be noted that $V \to +\infty$ and $\Delta_2/\Delta_1 \to +\infty$ have been assumed in the derivation; consequently, (12) is accurate only when V and Δ_2/Δ_1 are large. Putting (12) into (11), we find

$$\frac{a}{b} = 2^{1/3}\left(\frac{m}{n}\right)^{2/3}. \tag{13}$$

This result is independent of V and Δ_2/Δ_1 provided that they are sufficiently large. In the case of the fundamental mode, i.e., $m = n = 1$, we have $a/b = 2^{1/3} = 1.26$. To the author's knowledge, this property of the strip waveguide, namely that the E^x_{mn} and E^y_{mn} modes become degenerate with a particular aspect ratio, has not been previously discussed. Polarization-insensitive devices which require phase matching to be satisfied by both polarized modes can thus be constructed from isotropic strip waveguides.

III. Numerical Examples

To confirm the accuracy of (9), a semiconductor strip waveguide with $2a = 3$ μm, $2b = 1$ μm, $n_1 = 3.44$, $n_2 = 3.40$, and $n_3 = 1.0$ [10], [12], [13] operated at $\lambda = 1.15$ μm (corresponding to $V = 1.42893$) is analyzed. The normalized propagation constant P^2, defined by $P^2 = [(\beta_i/k)^2 - n_2^2)]/(n_1^2 - n_2^2)$, calculated from (9) is compared with those obtained from the effective-index method [1], [2], [5], [6], Marcatili's method [7], and the finite-difference method [12]. The results are presented in Table I for the E^x_{11}, E^y_{11}, and E_{11} (scalar) modes. As clearly shown by the data in Table I, the agreement between the results from the various methods is excellent. Equation (9) is accurate not only for the calculation of propagation constants but also for the calculation of modal birefringence. It has been shown both numerically [5] and analytically [15] that the effective-index method is highly accurate for a strip waveguide. The finite-difference solutions [12] in Table I are probably the poorest. The evidence comes from the observation that the same finite-difference method, when applied to a slab waveguide [12], overestimates the propagation constants by amounts comparable to the discrepancies between the finite-difference solutions and the effective-index solutions shown in Table I. (The finite-difference solutions for P^2 are 0.4319 and 0.3863 for the TE_0 and TM_0 modes of the slab waveguide [12], respectively, in contrast with the exact values 0.4273 and 0.3851.)

TABLE I
NORMALIZED PROPAGATION CONSTANT P^2 CALCULATED BY VARIOUS METHODS FOR THE E^x_{11}, E^y_{11}, AND E_{11} (SCALAR) MODES OF A STRIP WAVEGUIDE

Methods	E^x_{11}	E^y_{11}	E_{11} (Scalar)	Birefringence
Equation (9)	0.2939	0.2608	0.3030	0.0331
Effective-index	0.2939	0.2603	0.3026	0.0336
Marcatili	0.2938	0.2603	0.3025	0.0335
Finite-difference	0.2959	0.2617	0.3071	0.0342

$2a = 3$ μm, $2b = 1$ μm, $n_1 = 3.44$, $n_2 = 3.40$, $n_3 = 1.0$, $\lambda = 1.15$ μm.

Birefringence is the difference in P^2 between the E^x_{11} and E^y_{11} modes.

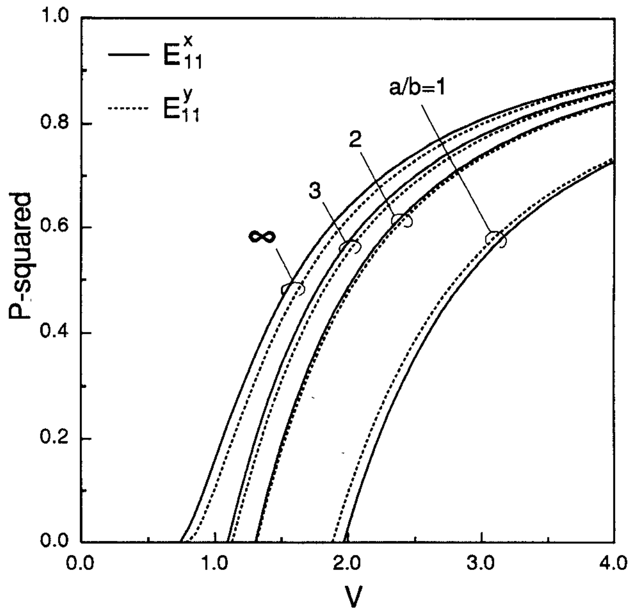

Fig. 2. Dispersion curves for the E^x_{11} and E^y_{11} modes of a strip waveguide with $n_1 = 3.44$, $n_2 = 3.40$, and $n_3 = 1.0$ for different values of aspect ratio a/b.

Equation (9) has been checked extensively by comparison with results from the effective-index method and Marcatili's method. In general, the results calculated from (9) agree very well with reference data over the whole guiding range of V. Significant discrepancies occur only in situations where the aspect ratio a/b and the index ratio Δ_2/Δ_1 become impracticably small. As an example, the dispersion curves for the E^x_{11} and E^y_{11} modes of the waveguide with the refractive indexes mentioned earlier are presented in Fig. 2 for various values of aspect ratio. The results obtained from (9) are hardly distinguishable from the effective-index solutions, which are therefore not shown in the figure. The aspect ratio required for achieving a specified modal birefringence for the fundamental mode is plotted in Fig. 3 as a function of V. As shown in this figure, the aspect ratio for zero birefringence, calculated from (11), decreases from about 1.85 toward 1.26 with increasing V, in agreement with (13). It is also obvious from Fig. 3 that the change in birefringence with aspect ratio decreases with increasing V. For a given positive birefringence, a minimum aspect ratio exists at a finite V.

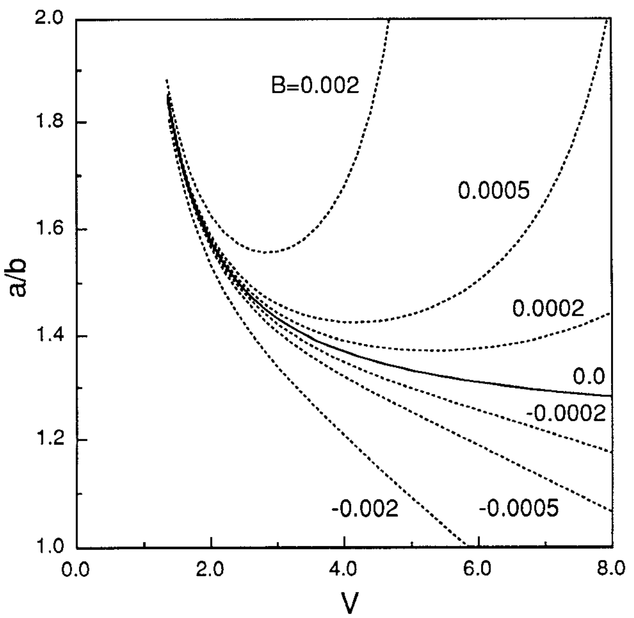

Fig. 3. Aspect ratio required for achieving a specified modal birefringence B in a strip waveguide with $n_1 = 3.44$, $n_2 = 3.40$, and $n_3 = 1.0$. B is the difference between the normalized propagation constants of the E_{11}^x and E_{11}^y modes.

IV. Conclusion

A simple and accurate relation has been derived to describe the dispersion characteristics of a strip waveguide. Apart from providing a much more efficient way for calculating dispersion, this relation brings out explicitly many physical properties of the waveguide and should be useful for the study and design of strip waveguides. A simple application of this relation has revealed an interesting property of the waveguide, namely, that it is always possible to make the two polarized modes of the waveguide degenerate by using an appropriate aspect ratio for the strip.

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References

[1] W. V. McLevige, T. Itoh, and R. Mittra, "New waveguide structures for millimeter-wave and optical integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 788–794, 1975.
[2] T. Itoh, "Inverted strip dielectric waveguide for millimeter-wave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol MTT-24, pp. 821–827, 1976.
[3] K. Ogusu, "Numerical analysis of the rectangular dielectric waveguide and its modifications," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 874–885, 1977.
[4] A. A. Oliner, S.-T. Peng, T.-I. Hsu, and A. Sanchez, "Guidance and leakage properties of a class of open dielectric waveguides: Part II —New physical effects," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 855–869, 1981.
[5] R. M. Knox and P. P. Toulios, "Integrated circuits for the millimeter through optical frequency region," in *Proc. MRI Symp. Submillimeter Waves*, 1970, pp. 497–516.
[6] K. S. Chiang, "Dual effective-index method for the analysis of rectangular dielectric waveguides," *Appl. Opt.*, vol. 25, pp. 2169–2174, 1986
[7] E. A. J. Marcatili, "Dielectric rectangular waveguide and directional coupler for integrated optics," *Bell Syst. Tech. J.*, vol. 48, pp. 2071–2102, 1969.
[8] R. Mittra, Y. L. Hou, and V. Jamnejad, "Analysis of open dielectric waveguides using mode-matching technique and variational methods," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 36–43, 1980.
[9] C. Yeh, K. Ha, S. B. Dong, and W. P. Brown, "Single-mode optical waveguides," *Appl. Opt.*, vol. 18, pp. 1490–1504, 1979.
[10] B. M. A. Rahman and J. B. Davies, "Vector-H finite element solution of GaAs/GaAlAs rib waveguides," *Proc. Inst. Elec. Eng.* pt. J, vol. 132, pp. 349–353, 1985.
[11] M. J. Robertson, S. Ritchie, and P. Dayan, "Semiconductor waveguides: Analysis of optical propagation in single rib structures and directional couplers," *Proc. Inst. Elec. Eng.* pt. J, vol. 132, pp. 336–342, 1985.
[12] M. S. Stern, "Semivectorial polarised finite difference method for optical waveguides with arbitrary index profiles," *Proc. Inst. Elec. Eng.* pt. J, vol. 135, pp. 56–63, 1988.
[13] P. C. Kendall, M. J. Adams, S. Ritchie, and M. J. Robertson, "Theory for calculating approximate values for the propagation constants of an optical rib waveguide by weighting the refractive index," *Proc. Inst. Elec. Eng.* pt. J, vol. 134, pp. 699–702, 1987
[14] A. W. Snyder and J. D. Love, *Optical Waveguide Theory*. London: Chapman and Hall, 1983, chs. 31–33
[15] K. S. Chiang, "Performance of the effective-index method for the analysis of dielectric waveguides," (submitted for publication).

Microwave Measurement of the Dielectric Constant of High-Density Polyethylene

Karlheinz Seeger

Abstract—**By applying a new microwave technique which involves observing interference fringes in transmission, metallizing the sample faces adjacent to the waveguide, and thus using the sample as a dielectric-filled metallic waveguide, the real part of the dielectric function of high-density polyethylene has been determined as 2.34 at room temperature and 2.29 at liquid nitrogen temperature (77 K) for the frequency range from 26.5 to 40 GHz.**

I. Introduction

Interest in the dielectric behavior of polymers at microwave frequencies has usually been focused on the loss tangent because of low-loss technical applications [1]. Not until quite recently has a precision method for the microwave measurement of the real part of the complex dielectric function been applied to semi-insulating semiconductors [2], [3]. In the present paper the real part of the dielectric constant, ϵ, of high-density polyethylene measured by this microwave interference technique will be reported. Since the loss tangent of this polymer is only of the order of 10^{-4}, the variation of ϵ over a microwave frequency band is of the same order of magnitude, which means that this variation can be neglected in the evaluation of an interference spectrum, quite similar to the case of high-resistivity semiconductors.

II. Experimental Technique

The experimental technique has been reported in detail in [2]. Therefore it suffices to explain it in principle only.

The sample with plane-parallel front and rear ends completely fills a rectangular waveguide for a length d. A Q-band waveguide for 26.5 to 40 GHz has been found convenient for

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